XVI. TANGENTI (DERIVATE) DEI PUNTI INTERNI

TANGENTI (derivate) DEI PUNTI INTERNI

Abbiamo visto nella ricerca precedente per gli insiemi come considerare il valore di $\cos^2\alpha_1 + \cos^2\alpha_2 < 1$, per ottenere dei punti interni. Vediamo quale è la derivata di quei punti.

Sia l’Ellisse:

$$\begin{cases} x = q\cos\alpha_1 \\ y = m\cos\alpha_2 \end{cases} \qquad \cos^2\alpha_1 + \cos^2\alpha_2 = d$$

dove il valore d è una costante compresa tra 0<d<1.Dunque possiamo anche scrivere $\cos\alpha_2 = \pm\sqrt{d-\cos^2\alpha_1}$ dove dovrà essere $d \geq \cos^2\alpha_1$ e quindi:

$$\begin{cases} x = q\cos\alpha_1 \\ y = m\left[\pm\sqrt{d-\cos^2\alpha_1}\right] \end{cases} \qquad \begin{cases} \dfrac{dx}{d\alpha_1} = -q\,\mathrm{sen}\,\alpha_1 \\ \dfrac{dy}{d\alpha_1} = m\left[\pm\sqrt{d-\cos^2\alpha_1}\right]' \end{cases}$$

La derivata di quest’ultima espressione è:

$$\frac{dy}{d\alpha_1} = m\frac{\cos\alpha_1\,\mathrm{sen}\,\alpha_1}{\pm\sqrt{d-\cos^2\alpha_1}}$$

$$\frac{dy}{dx}\frac{d\alpha_1}{d\alpha_1} = \frac{m}{q}\left[\frac{\cos\alpha_1\,\mathrm{sen}\,\alpha_1}{(-\mathrm{sen}\,\alpha_1)\left[\pm\sqrt{d-\cos^2\alpha_1}\right]}\right] = \frac{m}{q}\;\frac{\cos\alpha_1}{\pm\sqrt{d-\cos^2\alpha_1}} = \frac{m}{q}\frac{\cos\alpha_1}{\cos\alpha_2}$$

Dove per $\cos^2\alpha_1 + \cos^2\alpha_2 = 1$ si avrà $\dfrac{dy}{dx} = \dfrac{m}{q}\dfrac{1}{\tan\alpha_1}$ (derivata dell’Ellisse)

Per m=q=R è la tangente di una circonferenza.

Sia l’Iperbole:

$$\begin{cases} x = \dfrac{q}{\cos\alpha_1} \\ y = m\dfrac{\cos\alpha_1}{\cos\alpha_2} \end{cases}$$

da cui $\cos\alpha_2 = \pm\sqrt{d-\cos^2\alpha_1}$ $\qquad 0 < d < 1$

$$\begin{cases} \dfrac{dx}{d\alpha_1} = \dfrac{q\,\mathrm{sen}\,\alpha_1}{\cos^2\alpha_1} \\ \dfrac{dy}{d\alpha_1} = m\left[\dfrac{\pm\sqrt{d-\cos^2\alpha_1}}{\cos\alpha_1}\right]' \end{cases} \; ; \quad \frac{dy}{d\alpha_1} = m\left[\frac{\left[\pm\sqrt{d-\cos^2\alpha_1}\right]'\cos\alpha_1 - [\cos\alpha_1]'\left(\pm\sqrt{d-\cos^2\alpha_1}\right)}{\cos^2\alpha_1}\right]$$

La derivata dell’ultima espressione: $\left[\pm\sqrt{d-\cos^2\alpha_1}\right]' = \dfrac{\cos\alpha_1\,\mathrm{sen}\,\alpha_1}{\pm\sqrt{d-\cos^2\alpha_1}}$

$$\frac{dy}{d\alpha_1} = m\left[\frac{\cos^2\alpha_1\,\mathrm{sen}\,\alpha_1 + \mathrm{sen}\,\alpha_1\cos^2\alpha_2}{\cos^2\alpha_1\cos\alpha_2}\right]$$

$$\frac{dy}{dx}\ \frac{d\alpha_1}{d\alpha_1}=m\left[\frac{\cos^2\alpha_1\operatorname{sen}\alpha_1+\operatorname{sen}\alpha_1\cos^2\alpha_2}{\cos^2\alpha_1\cos\alpha_2}\right]\ \frac{\cos^2\alpha_1}{q\operatorname{sen}\alpha_1}\ =\ \frac{m}{q}\,\frac{\cos^2\alpha_1+\cos^2\alpha_2}{\cos\alpha_2}$$

Dove per $\cos^2\alpha_1+\cos^2\alpha_2=1$ si avrà $\frac{dy}{dx}=\frac{m}{q}\,\frac{1}{\operatorname{sen}\alpha_1}$ (derivata dell’Iperbole).

Siano le Parabole (p±x):

$$y=\frac{p}{1\mp\cos\beta_1}\cos\beta_2 \qquad\qquad x=\frac{p}{1\mp\cos\beta_1}\cos\beta_1$$

sapendo che $\cos\beta_2=\pm\sqrt{d-\cos^2\beta_1}$ e $\left[\pm\sqrt{d-\cos^2\beta_1}\right]'=\frac{\cos\beta_1\operatorname{sen}\beta_1}{\pm\sqrt{d-\cos^2\beta_1}}$

$$\frac{dy}{d\beta_1}=p\,\frac{\left[\pm\sqrt{d-\cos^2\beta_1}\right]'(1\mp\cos\beta_1)-(\operatorname{sen}\beta_1)\left(\pm\sqrt{d-\cos^2\beta_1}\right)}{(1\mp\cos\beta_1)^2\left(\pm\sqrt{d-\cos^2\beta_1}\right)}=$$

$$=p\,\frac{\operatorname{sen}\beta_1\cos\beta_1(1\mp\cos\beta_1)-\operatorname{sen}\beta_1\left(d-\cos^2\beta_1\right)}{(1\mp\cos\beta_1)\left(\pm\sqrt{d-\cos^2\beta_1}\right)}$$

$$\frac{dx}{d\beta_1}=p\,\frac{(-\operatorname{sen}\beta_1)(1\mp\cos\beta_1)-\cos\beta_1(+\operatorname{sen}\beta_1)}{(1\mp\cos\beta_1)^2}=p\,\frac{-\operatorname{sen}\beta_1}{(1\mp\cos\beta_1)^2}$$

$$\frac{dy}{dx}\,\frac{d\beta_1}{d\beta_1}=\frac{\cos\beta_1(1\mp\cos\beta_1)\mp\left(d-\cos^2\beta_1\right)}{-\left(\pm\sqrt{d-\cos^2\beta_1}\right)}=\frac{\cos\beta_1\mp d}{-\cos\beta_2}$$

da cui le due tangenti:

$$\frac{\cos\beta_1-d}{-\cos\beta_2}=\frac{d-\cos\beta_1}{\cos\beta_2} \qquad\qquad \frac{\cos\beta_1+d}{-\cos\beta_2}=-\frac{d+\cos\beta_1}{\cos\beta_2}$$

dove per d=1 si riavrebbero le tangenti delle due Parabole:

$$\frac{1-\cos\beta_1}{\operatorname{sen}\beta_1} \qquad\qquad -\frac{1+\cos\beta_1}{\operatorname{sen}\beta_1}$$